Device-Independent Certification of a Nonprojective Qubit Measurement

Esteban S. Gómez,1,2,3 Santiago Gómez,1,2,3 Pablo González,1,2,3 Gustavo Cañas,1,2,3 Johanna F. Barra,1,2,3 Aldo Delgado,1,2,3 Guilherme B. Xavier,2,3,4 Adán Cabello,5 Matthias Kleinmann,6 Tamás Vértesi,7 and Gustavo Lima1,2,3,*

1Departamento de Física, Universidad de Concepción, 160-C Concepción, Chile 2Center for Optics and Photonics, Universidad de Concepción, 160-C Concepción, Chile 3MSI-Nucleus for Advanced Optics, Universidad de Concepción, 160-C Concepción, Chile 4Departamento de Ingeniería Eléctrica, Universidad de Concepción, 160-C Concepción, Chile 5Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain 6Department of Theoretical Physics, University of the Basque Country UPV/EHU, P.O. Box 644, E-48080 Bilbao, Spain 7Institute for Nuclear Research, Hungarian Academy of Sciences, H-4001 Debrecen, P.O. Box 51, Hungary

(Received 27 June 2016; published 20 December 2016)

Quantum measurements on a two-level system can have more than two independent outcomes, and in this case, the measurement cannot be projective. Measurements of this general type are essential to an operational approach to quantum theory, but so far, the nonprojective character of a measurement can only be verified experimentally by already assuming a specific quantum model of parts of the experimental setup. Here, we overcome this restriction by using a device-independent approach. In an experiment on pairs of polarization-entangled photonic qubits we violate by more than 8 standard deviations a Bell-like correlation inequality that is valid for all sets of two-outcome measurements in any dimension. We combine this with a device-independent verification that the system is best described by two qubits, which therefore constitutes the first device-independent certification of a nonprojective quantum measurement.

DOI: 10.1103/PhysRevLett.117.260401

The qubit is the abstract notion for any system that can be modeled in quantum theory by a two-level system. In such a system, any observable has at most two eigenvalues and hence any projective measurement can have at most two outcomes. Still, a qubit allows for an infinite number of different two-outcome measurements, the value of which, in general, cannot be known to the observer beforehand, but rather follows a binomial distribution. In quantum information theory, additional properties reflecting this binary structure have been revealed; e.g., the information capacity of a qubit is one classical bit, even when using entangled structure have been revealed; e.g., the information capacity rather follows a binomial distribution. In quantum information, in general, cannot be known to the observer beforehand, but different two-outcome measurements, the value of which, hence any projective measurement can have at most two eigenvalues and a system, any observable has at most two eigenvalues and in nonorthogonal, but at the cost of allowing a third measurement outcome that indicates a failure of the discrimination procedure. The strategy with the lowest failure probability can be shown to be an irreducible three-outcome measurement [5]. Also recently, nonprojective measurements proved to be essential in purely information theoretical tasks like improving randomness certification [6].

A peculiarity of nonprojective qubit measurements with more than two irreducible outcomes is that there is no known way to implement them within a qubit system. Rather, the measurement apparatus needs to manifestly work outside of what would be modeled by a qubit alone. To some extent it is therefore a matter of perspective whether, at all, one is willing to admit such nonprojective measurements on a qubit system. However, device-independent self-testing [7] allows us to demonstrate that a qubit description is appropriate for the tested system, by showing that, with high precision, any measurement on the system can be modeled as a qubit measurement.

A key observation is that it is not possible to show that a measurement is irreducibly nonbinary insofar as we consider a single quantum system, as the outcomes of measurements on a single system can always be explained in terms of a hidden variable model where all $E^{(k)}_k$ are either 1 or 0 and $p_\lambda$ depends on the preparation of the system. The situation changes when considering the correlations between independent measurements on an entangled system [8], but still a violation of a conventional Bell
bipartite scenario, cf. Fig. 1(a), where one party, Alice, application of a nonbinary measurement on a qubit. We consider a lowered to 0.9845, enabling the device-independent certif-

ities, where qubit measurements with more than three outcomes outperform the maximal violation attainable when

the other party, Bob, chooses one among three measurements 

for Alice

pressed) and produces an outcome (represented by a light

flashing). All possible measurements have two outcomes, except

Alice’s measurement \(x = 3\) cannot have been a measurement composed of binary quantum measurements on whatever quantum system and selected by the measurement apparatus, as shown in Fig. 1(b).

Since projective measurements on a qubit necessarily are binary or trivial, a violation of \(I < 1.2711\) certifies the implementation of a nonprojective measurement. This requires, however, that the system at Alice’s laboratory is actually a qubit, which is manifestly the case in our experimental setup, as we explain below. In addition, this assertion of Alice’s system being a qubit can also be verified in a device-independent way by measuring the violation of the Clauser-Horne-Shimony-Holt (CHSH) Bell inequality [16]. If this violation is maximal, the joint state has to be a maximally entangled qubit-qubit state [17–19], independently of what measurement apparatuses are used. If the observed value for the CHSH violation deviates by \(\epsilon\) from the maximum \(2\sqrt{2} - 2\), the state must still have a fidelity of at least 1 − 2.2\(\epsilon\) with a maximally entangled qubit-qubit state [20]. A description of the system in the corresponding qubit-qubit-space is hence accurate up to 2.2\(\epsilon\).

The setup of our experiment is shown in Fig. 2. Degenerate 810 nm photon pairs, with orthogonal polarizations, are produced from spontaneous parametric downconversion (SPDC) in a bulk type-II nonlinear periodically poled potassium titanyl phosphate (PPKTP) 20 mm long crystal. The crystal is pumped by a single-longitudinal mode continuous wave 405 nm laser with 1 mW of optical power. We resort to an ultrabright source architecture, where the type-II nonlinear crystal is placed inside an intrinsically phase-stable Sagnac interferometer [21–23]. This interferometer is composed of two laser mirrors, a half-wave plate (HWP\(_2\)), and a polarizing beam splitter cube (PBS\(_1\)). The HWP\(_2\) and PBS\(_1\) are both dual wavelength with antireflection coatings at 405 and 810 nm. The fast axis of the HWP\(_2\) is set at 45 deg with respect to the horizontal, such that down-converted photons are generated in the clockwise and counterclockwise directions. The clockwise and counterclockwise propagating modes overlap inside the polarizing beam splitter and, by properly adjusting the pump beam polarization mode, the two-photon state emerging at the output ports is \(|\psi^+\rangle = (|HV\rangle + |VH\rangle)/\sqrt{2}\), where \(|H\rangle (|V\rangle)\) denotes the horizontal (vertical) polarization of a down-converted photon. Because of the phase-matching conditions, there may be entanglement between other degrees of freedom of the generated photons, or coupling between the polarization and the momentum of these photons that would compromise the quality of the polarization

\[
I = P(00|00) + P(00|11) + P(00|22) \\
- P(00|01) - P(00|12) - P(00|20) \\
- P(00|30) - P(10|31) - P(20|32). \tag{1}
\]

When restricted to binary quantum measurements, not necessarily on a qubit, then the value of \(I\) is upper bounded by 1.2711, cf. the Supplemental Material [12]. Without this restriction, the maximal quantum value of \(I\) is \(3\sqrt{3}/4 \approx 1.2990\) and can be achieved for two qubits using a maximally entangled state, cf. the Supplemental Material [12]. Thus, an experiment violating the inequality \(I < 1.2711\) proves that Alice’s measurement \(x = 3\) cannot have been a measurement composed of binary quantum measurements on whatever quantum system and selected by the measurement apparatus, as shown in Fig. 1(b).

Here, we introduce an inequality where this threshold is lowered to 0.9845, enabling the device-independent certification of a nonbinary measurement on a qubit. We consider a bipartite scenario, cf. Fig. 1(a), where one party, Alice, chooses one among four measurements \(x = 0, 1, 2, 3\) while the other party, Bob, chooses one among three measurements \(y = 0, 1, 2\). All measurements have two outcomes \(a = 0, 1\) and \(b = 0, 1\) except Alice’s measurement \(x = 3\), which has three outcomes \(a = 0, 1, 2\). We denote by \(P(ab|xy)\) the probability for outcome \(a\) and \(b\) when the setting \(x\) and \(y\) were chosen and consider the expression

\[
I = P(00|00) + P(00|11) + P(00|22) - P(00|01) - P(00|12) - P(00|20) - P(00|30) - P(10|31) - P(20|32). \tag{1}
\]
To avoid this we add extra spectral and spatial filtering. To remove the remaining laser light we adopt a series of dichroic mirrors followed by a longpass color glass filter. Then, Semrock high-quality (peak transmission greater than 90%) narrow bandpass (full width at half maximum of 0.5 nm) interference filters centered at 810 nm are used to ensure that phase-matching conditions are achieved with the horizontal and vertical polarization modes at degenerated frequencies.

The indistinguishability of the photon pair modes (“HV” and “VH”) is guaranteed by coupling the generated down-converted photons into single mode fibers. These fibers implement a spatial mode filtering of the down-converted light, destroying any residual spatial entanglement or polarization-momentum coupling. To maximize the source’s spectral brightness, we resort to a numerical model [24]. In our case, the beam waist \( w_p \) of the pump beam, and \( w_{SPDC} \) of the selected down-converted modes, at the center of the PPKTP crystal, are adjusted by using a 20 cm focal length lens \( L_1 \) and 10x objective lenses. The optimal condition for the maximal photon per yield is obtained when \( w_{SPDC} = \sqrt{2}w_p \) with \( w_p = 40 \mu m \). The observed source spectral brightness was 410000 photon pairs (s mW nm)\(^{-1}\). The quality of the polarization entanglement generated at the source site was measured by observing a mean two-photon visibility of 0.987 ± 0.002 while measuring over the logical and diagonal polarization bases. Because of the demand of a high overall visibility we built a coincidence electronics based on a field programmable gate array platform and capable of implementing up to 1 ns coincidence windows, thus reducing the probability for an accidental coincidence count to less than 0.00025. Therefore, the evaluation of the data does not require a separate treatment for accidental coincidence counts. The down-converted photons are registered using PerkinElmer single-photon avalanche detectors with an overall detection efficiency of 15%. We account for this by including the assumption into our analysis that the detected coincides are a fair sample from the set of all photon pairs.

Alice’s and Bob’s binary measurements are implemented using a set composed of a quarter-wave plate (QWP), a half-wave plate (HWP), and a polarizing beam splitter (PBS) for each party, cf. Fig. 2. A high-quality film polarizer is also used in front of the detectors (not shown for sake of clarity) to obtain a total extinction ratio of the polarizers equal to \( 10^7 : 1 \). Therefore, in our experiment the two-photon visibility is not upper limited by the polarization contrast of our measurement apparatuses. Alice’s three-outcome measurement \( x = 3 \) is implemented using a polarization based two-path Sagnac interferometer (depicted in Alice’s violet box). The elements of the three-outcome qubit measurement are defined by HWP\(_r\), HWP\(_l\), and HWP\(_o\). The coincidence counts between Alice’s and Bob’s detectors are recorded using a coincidence electronics unit based on a field programmable gate array device.
polarization transformations implemented with HWPs placed inside the interferometer, as shown in Fig. 2. The plate located at the clockwise reflected mode is denoted by HWP$_r$, and the plate at the counterclockwise transmitted mode by HWP$_l$. The fast axis of HWP$_r$ is oriented in the direction of the horizontal axis, while HWP$_l$ is oriented at an angle of 117.37°. The two propagation modes are then superposed again at the PBS, and at one of the output ports of the interferometer an extra HWP$_o$, oriented at 112.5°, and a PBS are used to conclude the three-outcome measurement. Further details on the implementation of Alice’s three-outcome measurement $x = 3$ are given in the Supplemental Material [12].

In the experiment, the correlations $P(ab|xy)$ in I were measured by integrating coincidences over a time of 240 s for each outcome and normalizing over the total number of coincidences per setting. The results are shown in Fig. 3 and yield a measured value of $I = 1.2824 \pm 0.0013$.

The measurement settings were implemented independently for Alice and Bob, justifying the assumption that Alice’s measurements also act independently of Bob’s measurement setting $y$ and vice versa. Hence, any explanation in terms of binary measurements on an arbitrary quantum system is excluded by 8.7 standard deviations, which corresponds to a $p$ value of $1.6 \times 10^{-18}$.

In order to prove that Alice’s measurement $x = 3$ is a nonprojective quantum measurement, we need also to verify that Alice’s system can be properly described as a qubit. We rely here on two complementary arguments. First, one can resort to the design of the experiment where the source is designed to produce entanglement in polarization, i.e., qubit-qubit entanglement. Second, we measured the CHSH correlations with our setup and observed a violation of $2\sqrt{2} - 2 - \epsilon$ with $\epsilon = 0.0253 \pm 0.0014$ and hence the fidelity with a maximally entangled qubit-qubit state is guaranteed in a device-independent way to be at least 0.9351 within 3 standard deviations [20]. Note that we measured the CHSH correlations using the same source and the same measurement setup as we used for the measurement of $I$—except that different angles at the HWPs are adopted. Except for some ubiquitous adversary ad hoc models we can hence conclude that also in the measurement of $I$ the fidelity of the state with a maximally entangled qubit-qubit state is at least 0.9351. Notice that the estimate for the fidelity is pessimistic since imperfections in the measurement apparatuses reduce the CHSH violation and therefore lower the bound on the fidelity. Still, in an alternative explanation where 93.51% of the times binary measurement was used, a bound of $I < 0.9351 \times 1.2711 + 0.0649 \times 3\sqrt{3}/4 < 1.2730$ would have to be obeyed, which is clearly violated in the experiment.

Our result shows that three-outcome nonprojective measurements can produce strictly stronger correlations between two qubits than projective two-outcome measurements on any quantum system and, therefore, that nature cannot be described in terms of binary quantum tests, not even when these tests are performed on two-level quantum systems. Quantum theory predicts also qubit-qubit correlations that cannot be explained as produced by three-outcome measurements. Observing them requires an overall visibility above 0.9927, which is beyond what is currently feasible in our setup (cf. the Supplemental Material [12]). Further theoretical and experimental efforts will be needed to identify and produce qubit correlations, which can only be explained by four-outcome nonprojective measurements. This will be the furthest we can go, as qubit correlations can always be accounted for that way [25].

We thank Antonio Acín, Marcelo Terra-Cunha, Paolo Mataloni, Valerio Scarani, Jaime Carñí, Miguel Solís-Prosser, and Omar Jiménez for conversations and assistance. This work was supported by FONDECYT Grants No. 1160400, No. 11150325, No. 11150324,
No. 1140635, and No. 1150101, Milenio Grant No. RC130001, PIA-CONICYT Grant No. PFB0824, OTKA Grant No. K111734, the FQXi large grant project “The Nature of Information in Sequential Quantum Measurements”, the project FIS2014-60843-P “Advanced Quantum Information” (MINECO, Spain) with FEDER funds, the Knut and Alice Wallenberg Foundation, Sweden (Project “Photonic Quantum Information”), the EU (ERC Starting Grant GEDENTQOPT), and the DFG (Forschungsstipendium KL 2726/2-1). P.G. and J. F. B. acknowledge the financial support of CONICYT.

*glima@udec.cl


