Compact set of invariants characterizing graph states up to eight qubits

Adán Cabello¹, Antonio J. López-Tarrida¹, Pilar Moreno¹, José R. Portillo²

¹Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain
²Departamento de Matemática Aplicada I, Universidad de Sevilla, E-41012 Sevilla, Spain

e-mail: tarrida@us.es

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Graph states [1, 2] are fundamental in quantum information, specially in quantum error-correction [3, 4, 5] and measurement-based quantum computation [6]. Graph states also play a fundamental role in the study of entanglement. Two quantum states have the same entanglement if and only if they are equivalent under local unitary (LU) operations. For \( n \geq 4 \) there is an infinite amount of different entanglement (i.e., LU-inequivalent) classes. However, if we focus on graph states, then the discussion becomes simpler. Previous results suggest that for graph states of \( n < 27 \) qubits, the notion of LU-equivalence and LC-equivalence coincide [7].

The aim of our work is to solve the following problem: given an \( n \)-qubit graph state with \( n < 9 \) qubits, deciding which entanglement class it belongs to just by examining some of the state's intrinsic properties (i.e., without generating the whole LC-class). The solution to this problem is of practical importance: if one needs to prepare a graph state \( |G\rangle \) and knows that it belongs to one specific class, then one can prepare \( |G\rangle \) by preparing the LC-equivalent state \( |G'\rangle \) requiring the minimum number of entangling gates and the minimum preparation depth of that class (see [1, 2, 8]), and then transform \( |G'\rangle \) into \( |G\rangle \) by means of simple one-qubit unitary operations. The problem is that, so far, no simple set of invariants which distinguishes between all classes of entanglement is known, even for graph states with \( n \leq 7 \) qubits.

The set of entanglement measures proposed in [1, 2] fails to distinguish between inequivalent classes (i.e., between different types of entanglement). Therefore, we cannot use these invariants for deciding which entanglement class a given state belongs to.

Van den Nest, Dehaene, and De Moor (VDD) have proposed a finite set of invariants that characterizes all classes [9]. However, already for \( n = 7 \), this set has more than \( 2 \times 10^{36} \) invariants which are not explicitly calculated anywhere, so this set is not useful for classifying a given graph state. In our work we show that, if \( n \leq 8 \), then 4 invariants related to those of VDD are enough to recognize the type of entanglement.

We introduce a new type of LC-invariant, the multiplicity \( M(A_\omega) \), that counts the number of support-related classes with the same cardinality. We have shown that only 4 of these multiplicities are enough to decide which entanglement class a graph state of \( n \leq 8 \) qubits belongs to: the multiplicities of some of the VDD’s invariants of values 0, 1, 3, and 4. Therefore, these 4 LC-invariants characterize any LC-class of \( n \leq 8 \) qubits. A compact set of invariants that characterize all inequivalent classes of graph states with a higher number of qubits can be obtained by applying the same strategy. This can be done numerically up to \( n = 12 \), a number of qubits beyond the present experimental capability in the preparation of graph states [10].

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References